NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS (EXTENSION 1)

2014 HSC Course Assessment Task 3 (Trial Examination)
Tuesday June 24, 2014

General instructions

- Working time – 2 hours. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer sheet provided (numbered as page 13)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED:** .....

Class (please ✔)

◯ 12M3A – Mr Zuber
◯ 12M3B – Mr Berry
◯ 12M3C – Mr Lowe
◯ 12M4A – Ms Ziaziaris
◯ 12M4B – Mr Lam
◯ 12M4C – Mr Ireland

Marker's use only.

QUESTION	1-10	11	12	13	14	Total	%
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{70}$	

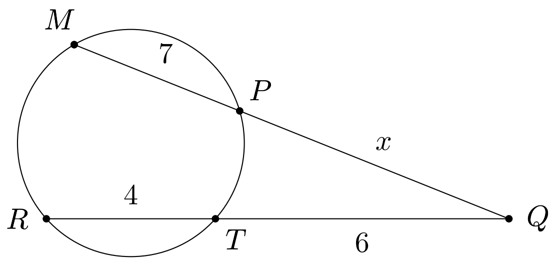
Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer sheet provided.

Questions **Marks**

1. What is the value of x if $MP = 7$, $RT = 4$, $TQ = 6$? **1**



(A) $\dfrac{21}{2}$

(B) 5

(C) $\dfrac{24}{7}$

(D) 9

2. What is the remainder when $8x^3 + 4x^2 - 3x - 8$ is divided by $x - 1$? **1**

(A) -1 (B) 1 (C) -9 (D) -8

3. Which of the following calculations is required to calculate the acute angle between the lines $x + y = 1$ and $2x - y = 1$? **1**

(A) $\tan\theta = \left|\dfrac{-1+2}{1+2}\right|$

(B) $\tan\theta = \left|\dfrac{-1+2}{2}\right|$

(C) $\tan\theta = \left|\dfrac{-1-2}{1-2}\right|$

(D) $\tan\theta = \left|\dfrac{-1-2}{-2}\right|$

4. What is the general solution to $2\sin\dfrac{x}{2} = -1$? **1**

(A) $2n\pi - (-1)^n\dfrac{\pi}{3}$

(B) $n\pi - (-1)^n\dfrac{\pi}{6}$

(C) $n\pi - (-1)^n\dfrac{\pi}{3}$

(D) $2n\pi - (-1)^n\dfrac{\pi}{6}$

5. Solve: $x^2\left(x^2-9\right) - 10 > 0$. **1**

(A) $-\sqrt{10} < x < \sqrt{10}$

(B) $x > \sqrt{10}, -1 < x < 1, x < -\sqrt{10}$

(C) $1 < x < \sqrt{10}, -\sqrt{10} < x < -1$

(D) $x > \sqrt{10}, x < -\sqrt{10}$

6. Which of the following integrals is equal to $\displaystyle\int \sin^2 4t\,dt$? **1**

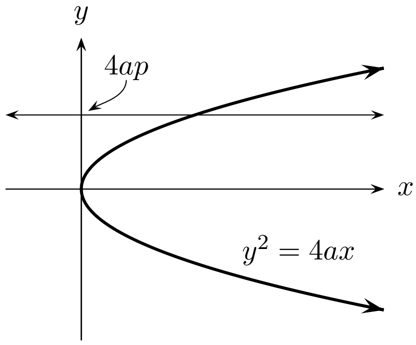
(A) $\displaystyle\int \frac{1+\cos 8t}{2}\,dt$

(B) $\displaystyle\int \frac{1-\cos 8t}{2}\,dt$

(C) $\displaystyle\int \frac{1+\cos 4t}{2}\,dt$

(D) $\displaystyle\int \frac{1-\cos 4t}{2}\,dt$

7. Which of the following integrals gives the volume of the solid formed when the area bound by $y^2 = 4ax$, the y axis and the line $y = 4ap$ is rotated about the y axis? **1**



(A) $\pi \displaystyle\int_0^{4ap} 4ax\,dx$

(B) $\pi \displaystyle\int_0^{4ap} \frac{y^4}{16a^2}\,dx$

(C) $\pi \displaystyle\int_0^{4ap} \frac{y^2}{16a}\,dy$

(D) $\pi \displaystyle\int_0^{4ap} \sqrt{4ax}\,dx$

8. Evaluate: $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{2x^2}$. **1**

(A) $\frac{1}{2}$

(B) 1

(C) 2

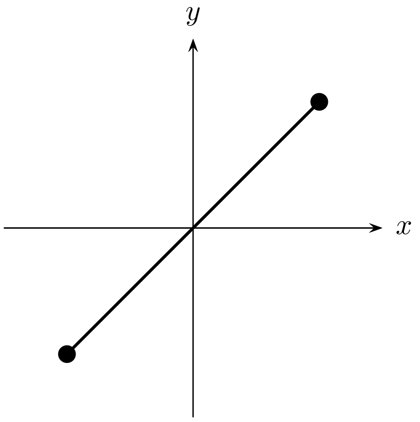
(D) ∞

9. A box contains 4 coins, one of which has two heads (no tails). If a coin is selected at random and then tossed, what is the probability that heads will show? **1**

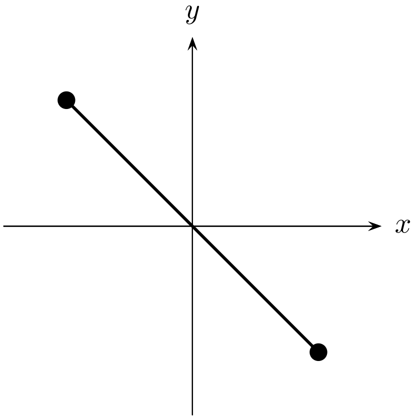
(A) $\frac{3}{8}$ (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{4}{5}$

10. Which of the following graphs represents $y = \cos\left(\cos^{-1} x\right)$? **1**

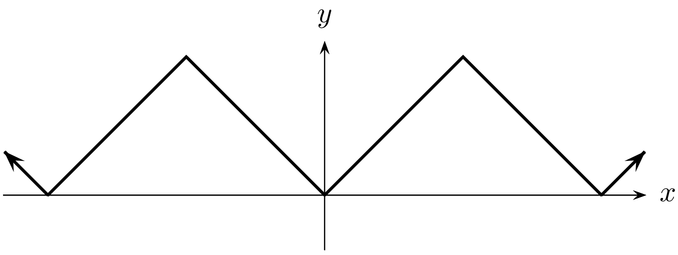
(A)



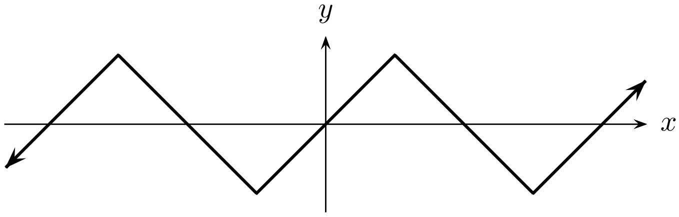
(B)



(C)



(D)



Examination continues overleaf. . .

Section II

60 marks
Attempt Questions 11 to 14
Allow approximately 1 hour and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Commence a NEW page. **Marks**

(a) Differentiate $\cos^{-1}\dfrac{x}{3}$. **1**

(b) Evaluate the following integrals: **1**

i. $\displaystyle\int \frac{1}{\cos^2 x}\,dx$.

ii. $\displaystyle\int_0^{\frac{1}{2}} \frac{2\,dx}{1+4x^2}$. **3**

(c) Evaluate $\displaystyle\int x\sqrt{1-x}\,dx$ by using the substitution $u = 1-x$. **3**

(d) If α, β and γ are roots of the equation $x^3 - 3x^2 - 5x - 7 = 0$, evaluate:

i. $\alpha+\beta+\gamma$. **1**

ii. $\alpha\beta\gamma$. **1**

iii. $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$. **2**

(e) If $\cos x\cos y = \dfrac{1}{4}\left(\sqrt{3}-\sqrt{2}\right)$ and $\sin x\sin y = \dfrac{1}{4}\left(\sqrt{3}+\sqrt{2}\right)$, find the smallest positive value of $(x+y)$ in radians. **3**

Question 12 (15 Marks) Commence a NEW page. **Marks**

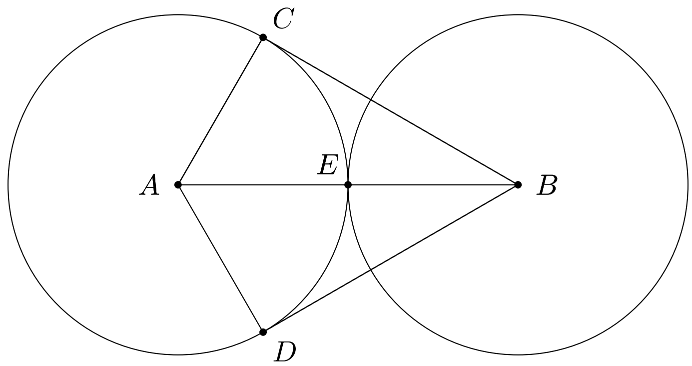
(a) Prove that $\tan^{-1}\dfrac{1}{4} + \tan^{-1}\dfrac{3}{5} = \dfrac{\pi}{4}$. **3**

(b) i. Express $\sqrt{2}\cos x - \sqrt{2}\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **2**

ii. Hence sketch $y = \sqrt{2}\cos x - \sqrt{2}\sin x$ for $0 \le x \le 2\pi$. **2**

iii. Hence find the value of k for which $\sqrt{2}\cos x - \sqrt{2}\sin x = k$ will have 3 solutions in the domain $0 \le x \le 2\pi$. **1**

(c) Two circles of *equal radii* and centres A and B respectively, touch externally at E. BC and BD are tangents from B to the circle with centre A.



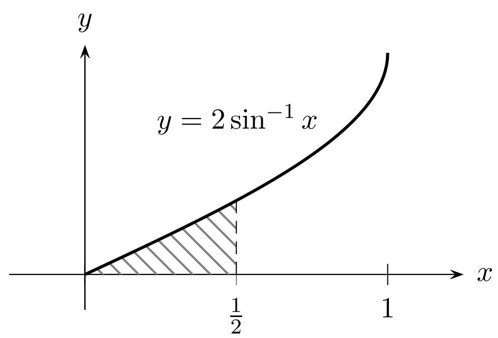
Copy or trace the diagram into your working booklet.

i. Show that $BCAD$ is a cyclic quadrilateral. **2**

ii. Show that E is the centre of the circle which passes through B, C, A and D. **1**

iii. Show that $\angle CBA = \angle DBA = 30°$. **1**

(d) The diagram below shows $f(x) = 2\sin^{-1} x$. The area enclosed between the curve, the line $x = \frac{1}{2}$ and the x axis is shaded.



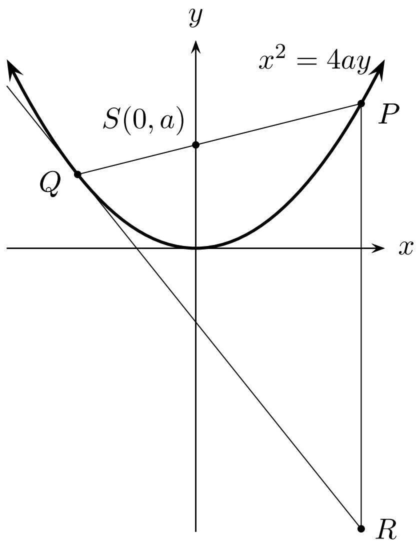
i. Find $f\left(\dfrac{1}{2}\right)$. **1**

ii. Show that the area of the shaded region is given by **2**

$$\frac{\pi}{6} + \sqrt{2} - 2 \text{ units}^2$$

Question 13 (15 Marks) Commence a NEW page. **Marks**

(a) PQ is a focal chord of the parabola $x^2 = 4ay$, where $P(2ap, ap^2)$ and $Q(2aq, aq^2)$. QR is the tangent at Q and RP is parallel to the axis of the parabola.



i. Show that $pq = -1$. **2**

ii. Find the equation of the locus of R. **3**

(b) The acceleration of a particle is given by $\ddot{x} = 4x - 4$. Initially, $x = 6$ and $|v| = 8$.

i. Show that $v^2 = 4x^2 - 8x - 32$. **2**

ii. Find the set of possible values of x. **1**

(c) An object is removed from a freezer at $-5°$C and is placed in a room where the temperature is kept constant at 15°C. The temperature of the object T then changes according to the rule

$$\frac{dT}{dt} = k(15 - T)$$

where t is in minutes and k is a constant.

i. Verify that $T = 15 - Ae^{-kt}$ satisfies this condition. **1**

ii. Find the value of A. **1**

iii. If initially the temperature of the object was increasing at 5°C per minute, find the value of k. **1**

iv. Find, correct to the nearest second, the time it takes for the object's temperature to rise to 0°C. **1**

(d) Prove by induction that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers n. **3**

Question 14 (15 Marks) Commence a NEW page. **Marks**

(a) The position of a particle moving along the x axis is given by $x = t^2 e^{2-t}$, where x is in metres and t is in seconds respectively.

i. Find an expression for the velocity in terms of time. **1**

ii. Show that the particle is initially at rest. **1**

iii. Find the time for which the particle is next at rest. **1**

iv. What happens as time increases indefinitely? **1**

(b) It is know that a solution exists near $x = 2$ for the equation $\log_e x = (x-1)^2$. **2**

Use one application of Newton's Method to find a better approximation to $x = 2$, correct to two decimal places.

(c) Consider the function $y = \log_e \dfrac{2x}{2+x}$.

i. Show that the domain of the function is **2**

$$D = \{x : x < -2, x > 0\}$$

ii. Find the value of x for which $y = 0$. **1**

iii. Show that $\dfrac{dy}{dx} = \dfrac{2}{x(2+x)}$ and hence show that the function is increasing for all x in the domain. **2**

iv. Are there any points of inflexion? Justify your answer. **2**

v. Sketch the graph of the function. **2**

End of paper.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE: $\ln x = \log_e x, \; x > 0$

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

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1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ
6 – Ⓐ Ⓑ Ⓒ Ⓓ
7 – Ⓐ Ⓑ Ⓒ Ⓓ
8 – Ⓐ Ⓑ Ⓒ Ⓓ
9 – Ⓐ Ⓑ Ⓒ Ⓓ
10 – Ⓐ Ⓑ Ⓒ Ⓓ

EXT 1 MATHEMATICS 2014 HSC ASSESSMENT TASK 3 - SOLUTIONS

1. B 2. B 3. C 4. A 5. D 6. B 7. B 8. A 9. C 10. A

1. $(x+7)x = 10 \times 6$

$x^2+7x-60=0$

$(x+12)(x-5)=0$

$x=5$

2. $8(1)+4(1)-3(1)-8$

$=1$

3. $\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\dfrac{-1-2}{1+(-1)(2)}\right|$

$= \left|\dfrac{-1-2}{1-2}\right|$

4. $\sin\dfrac{x}{2} = -\dfrac{1}{2}$

$\dfrac{x}{2} = n\pi + (-1)^n\left(-\dfrac{\pi}{6}\right)$

$x = 2n\pi - (-1)^n\left(\dfrac{\pi}{3}\right)$

5. $x^2(x^2-9)-10>0$

$x^4-9x^2-10>0$

$(x^2-10)(x^2+1)>0$

$(x-\sqrt{10})(x+\sqrt{10})(x^2+1)>0$

$x>\sqrt{10},\ x<-\sqrt{10}$

6. $\displaystyle\int \frac{1-\cos 8t}{2}\,dt$

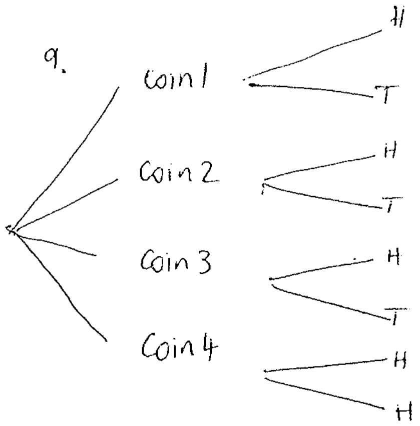
7. $\displaystyle\pi\int_0^{4ap} x^2\,dy$

$\displaystyle\pi\int_0^{4ap} \left(\frac{y^2}{4a}\right)^2 dy$

8. $\displaystyle\lim_{x\to 0} \frac{\sin x \sin x}{2x \cdot x}$

$=\dfrac{1}{2}$

9.



$\dfrac{5}{8}$

10. Plot points

$\cos(\cos^{-1} 1) = 1$

$\cos(\cos^{-1} -1) = -1$

Restricted because $\cos^{-1}$ first.

11. (a) Let $y = \cos^{-1}\left(\frac{x}{3}\right)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{9-x^2}}$$

(b) (i) $\displaystyle\int \frac{1}{\cos^2 x}\,dx$

$= \displaystyle\int \sec^2 x\,dx$

$= \tan x + C$

(ii) $\displaystyle\int_0^{\frac{1}{2}} \frac{2\,dx}{1+4x^2}$

$= 2\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{1+4x^2}$

$= \frac{2}{4}\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{\frac{1}{4}+x^2}$

$= \frac{1}{2}\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{\left(\frac{1}{2}\right)^2+x^2}$

$= \frac{1}{2}\times 2\left[\tan^{-1}2x\right]_0^{\frac{1}{2}}$

$= \tan^{-1}1 - \tan^{-1}0$

$= \frac{\pi}{4}$

OR $\displaystyle 2\int_0^{\frac{1}{2}} \frac{dx}{1+4x^2}\,dx$

$= 2\times\frac{1}{2}\left[\tan^{-1}2x\right]_0^{1/2}$

(c) $\displaystyle\int x\sqrt{1-x}\,dx$

$u = 1-x$

$\frac{du}{dx} = -1$

$du = -dx$

$= -\displaystyle\int (1-u)\,u^{1/2}\,du$

$= -\displaystyle\int \left(u^{1/2} - u^{3/2}\right)du$

$= \displaystyle\int \left(u^{3/2} - u^{1/2}\right)du$

$= \frac{2}{5}u^{5/2} - \frac{2u^{3/2}}{3} + C$

$= \frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + C$

(d) $x^3 - 3x^2 - 5x - 7 = 0$

(i) $\alpha + \beta + \gamma = 3$

(ii) $\alpha\beta\gamma = 7$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-5}{7}$

(e) $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \frac{1}{4}(\sqrt{3} - \sqrt{2}) - \frac{1}{4}(\sqrt{3} + \sqrt{2})$

$\cos(x+y) = -\frac{1}{2}\sqrt{2}$

$\cos(x+y) = -\frac{1}{\sqrt{2}}$

$\therefore x+y = \pi - \frac{\pi}{4}$

$x+y = \frac{3\pi}{4}$

12, a) Prove $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{3}{5} = \frac{\pi}{4}$

Let $\tan^{-1}\frac{1}{4} = \alpha$ and $\tan^{-1}\frac{3}{5} = \beta$

$\therefore \tan\alpha = \frac{1}{4}$ $\quad \tan\beta = \frac{3}{5}$

$\therefore$ Prove $\alpha + \beta = \pi/4$

Now, $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \dfrac{\frac{1}{4} + \frac{3}{5}}{1 - \frac{1}{4}\cdot\frac{3}{5}}$

$= \dfrac{\frac{17}{20}}{\frac{17}{20}}$

$\tan(\alpha+\beta) = 1$

$\therefore \alpha+\beta = \tan^{-1} 1$

$= \pi/4$

$\therefore \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{3}{5} = \frac{\pi}{4}$

(b) (i) $\sqrt{2}\cos x - \sqrt{2}\sin x = R\cos(x+\alpha)$

$R = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$

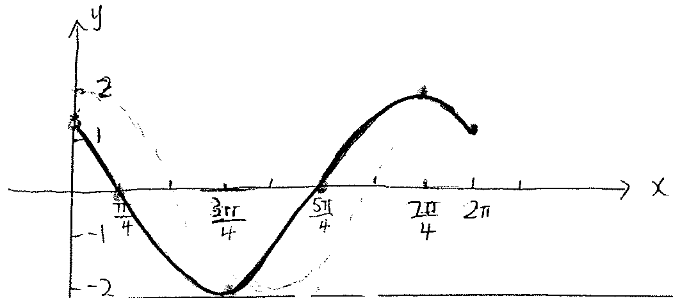
$\boxed{R = 2}$

$\alpha = \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}}$

$\alpha = \tan^{-1} 1$

$\boxed{\alpha = \frac{\pi}{4}}$

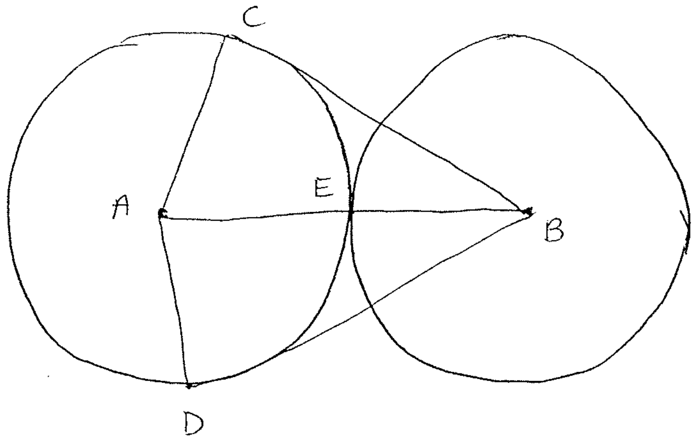
(ii) $\sqrt{2}\cos x - \sqrt{2}\sin x = 2\cos\left(x + \frac{\pi}{4}\right)$



Endpoints 1 mark.
Max/Min 1 mark.

(iii) $R = \sqrt{2}$

(c)



(i) $\angle ACB = 90^\circ$ (radius is perpendicular to tangent at point of contact)

Sim, $\angle ADB = 90^\circ$

Since opposite angles $\angle ACB$, $\angle ADB$ are supplementary,

then ADBC is cyclic.

(ii) AEB is a straight line - line joining the centres of 2 circles passes through their point of contact.

$\angle ADB = 90^\circ$

$\therefore$ AB is a diameter ($\angle$ in a semicircle is a rightangle)

$\therefore AE = EB = DE = CE$.

(iii) ΔADE is equilateral as all sides equal from (ii)

$\therefore \angle DAE = 60^\circ$

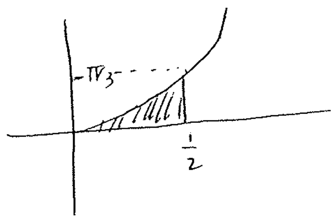
$\therefore \angle DBA = 30^\circ$ (angle sum of $\Delta ABD = 180^\circ$)

Sim $\angle CBA = 30^\circ$.

(d) (i) $f(\frac{1}{2}) = 2\sin^{-1}(\frac{1}{2})$

$= 2 \times \frac{\pi}{6}$

$= \frac{\pi}{3}$



(ii) $\frac{y}{2} = \sin^{-1} x$

$\therefore \sin\frac{y}{2} = x$

$A = \text{Area of rectangle} - \int_0^{\pi/3} x\, dy$

$= \left(\frac{\pi}{3} \times \frac{1}{2}\right) - \int_0^{\pi/3} \sin\frac{y}{2}\, dy$

$= \frac{\pi}{6} + 2\left[\cos\frac{y}{2}\right]_0^{\pi/3}$

$\frac{\pi}{6} + 2\left[\cos\frac{\pi}{6} - \cos 0\right]$

$= \frac{\pi}{6} + 2\left(\frac{\sqrt{3}}{2} - 1\right)$

$= \left(\frac{\pi}{6} + \sqrt{3} - 2\right) u^2$

Area of rect = 1 mark

Integration = 1 mark

13. (a) (i) $m_{PQ} = \dfrac{aq^2 - ap^2}{2aq - 2ap}$

$= \dfrac{a(q-p)(q+p)}{2a(q-p)}$

$= \dfrac{q+p}{2}$

Equ'n PQ : $y - ap^2 = \dfrac{(p+q)}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap^2 - apq$

$y = \dfrac{(p+q)}{2}x - apq$

But PQ is a focal chord

$\therefore (0, a)$ satisfies equ'n.

$a = 0 - apq$

$\therefore pq = -1$

(ii) Equ'n QR

$y = \dfrac{x^2}{4a}$

$\dfrac{dy}{dx} = \dfrac{x}{2a}$

At $x = 2aq$

$m = q$

Equ'n QR : $y - aq^2 = q(x - 2aq)$

$y - aq^2 = qx - 2aq^2$

$y = qx - aq^2$ — ①

Coords R : Sub $x = 2ap$ into — ①

$y = 2apq - aq^2$

But $pq = -1$

$y = -2a - aq^2$

$\therefore R\left(2ap, -2a - aq^2\right)$

ie. $R\left(-\dfrac{2a}{q}, -2a, -aq^2\right)$

Locus R : $x = -\dfrac{2a}{q}$

Sub $q = -\dfrac{2a}{x}$ into y

$y = -2a - aq^2$

$y = -2a - a\left(-\frac{2a}{x}\right)^2$

$y = -2a - a\left(\frac{4a^2}{x^2}\right)$

$\boxed{y = -2a - \frac{4a^3}{x^2}}$

(b) $\ddot{x} = 4x - 4$

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4x - 4$

$\frac{1}{2}v^2 = \int (4x-4)\, dx$

$\frac{1}{2}v^2 = 2x^2 - 4x + C$

$x = 6,\ v = 8$

$32 = 2 \times 36 - 4 \times 6 + C$

$32 = 72 - 24 + C$

$-16 = C$

$\therefore \frac{1}{2}v^2 = 2x^2 - 4x - 16$

$v^2 = 4x^2 - 8x - 32$

(ii) $4x^2 - 8x - 32 \geqslant 0$

$x^2 - 2x - 8 \geqslant 0$

$(x-4)(x+2) \geqslant 0$

$x \leqslant -2,\ x \geqslant 4$ OR $x \geqslant 4$ only as initially $x = 6$.

(c) (i) $LHS = \frac{dT}{dt}$

$= Ake^{-kt}$

$RHS = k(15 - T)$

$= k(15 - (15 - Ae^{-kt}))$

$= k(15 - 15 + Ae^{-kt})$

$= Ake^{-kt}$

$LHS = RHS.$

(ii) $T = 15 - Ae^{-kt}$

$t = 0,\ T = -5$

$-5 = 15 - Ae^{0}$

$\boxed{A = 20}$

(iii) $\frac{dT}{dt} = 5$ when $t = 0$

$\frac{dT}{dt} = Ake^{-kt}$

$5 = 20ke^{0}$

$k = \frac{1}{4}$

(iv) $0 = 15 - 20e^{-0.25t}$

$\frac{-15}{-20} = e^{-0.25t}$

$\frac{3}{4} = e^{-0.25t}$

$\ln\frac{3}{4} = -0.25t$

$t = -4\ln\frac{3}{4}$

$t = 1.15$ min OR 1min 9s.

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d) $9^{n+2} - 4^{n}$ is div. by 5.

Step 1: Show true for $n=1$

$9^{3} - 4^{1} = 725$

725 is div. by 5

$\therefore$ True for $n=1$

Step 2: Assume true for $n=k$.

ie. $9^{n+2} - 4^{n}$ is div. by 5.

ie. $9^{k+2} - 4^{k} = 5M$ for M a pos. integer.

Step 3: Prove true for $n=k+1$

ie. $9^{k+3} - 4^{k+1}$ is div. by 5.

$9.9^{k+2} - 4.4^{k} = 9(5M + 4^{k}) - 4.4^{k}$ (From step 2 $9^{k+2} = 5M + 4^{k}$)

$= 45M + 9.4^{k} - 4.4^{k}$

$= 45M + 5.4^{k}$

$= 5(9M + 4^{k})$

This is integral for M, k pos. int.

14. (a) $x = t^2 e^{2-t}$

(i) $v = e^{2-t} \cdot 2t + t^2 \cdot (-1) e^{2-t}$

$= 2te^{2-t} - t^2 e^{2-t}$

(ii) At rest initially then $t=0$, $v=0$

Sub $t=0$

$v = 0 - 0$

$v = 0$

(iii) At rest $v=0$

$0 = 2te^{2-t} - t^2 e^{2-t}$

$0 = te^{2-t}(2-t)$

$\therefore t=0, t=2$

Next at rest when $t=2$

(iv) As $t \to \infty$

$x = t^2 e^{2-t}$

$= \dfrac{t^2}{e^{2-t}}$

ie $x \to 0$

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b) Let $f(x) = \log_e x - (x-1)^2$

$f'(x) = \frac{1}{x} - 2(x-1)$

$f(2) = \ln 2 - 1$

$f'(2) = \frac{1}{2} - 2 = -\frac{3}{2}$

$a = a_1 - \dfrac{f(a_1)}{f'(a_1)}$

$= 2 - \dfrac{(\ln 2 - 1)}{-\frac{3}{2}}$

$= 2 + \dfrac{2(\ln 2 - 1)}{3}$

$= 1.795$

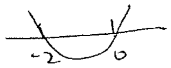
$= 1.80$

(c) $y = \log_e\left(\frac{2x}{2+x}\right)$

(i) $\frac{2x}{2+x} > 0$

$\frac{2x(2+x)^2}{2+x} > 0\,(2+x)^2$

$2x(2+x) > 0$



$\boxed{x < -2,\ x > 0}$

(ii) $0 = \log_e\left(\frac{2x}{2+x}\right)$

$e^0 = \frac{2x}{2+x}$

$2 + x = 2x$

$\boxed{2 = x}$

(iii) $y = \log_e 2x - \log_e(2+x)$

$\frac{dy}{dx} = \frac{2}{2x} - \frac{1}{2+x}$

$= \frac{2+x-x}{x(2+x)}$

$= \frac{2}{x(2+x)}$

For $x > 0$: $\frac{dy}{dx}$ is always positive

For $x < -2$: $\frac{2}{x(2+x)}$ is always positive.

$\underbrace{x}_{\text{negative}}\underbrace{(2+x)}_{\text{negative}}$

$\therefore \frac{dy}{dx} > 0$ for $x > 0,\ x < -2$.

(iv) Inflexion Pts : $\frac{d^2y}{dx^2} = 0$ & change in concavity

$\frac{d^2y}{dx^2} = \frac{x(2+x).0 - 2.(2+2x)}{x^2(2+x)^2}$

$= \frac{-4(1+x)}{x^2(2+x)^2}$

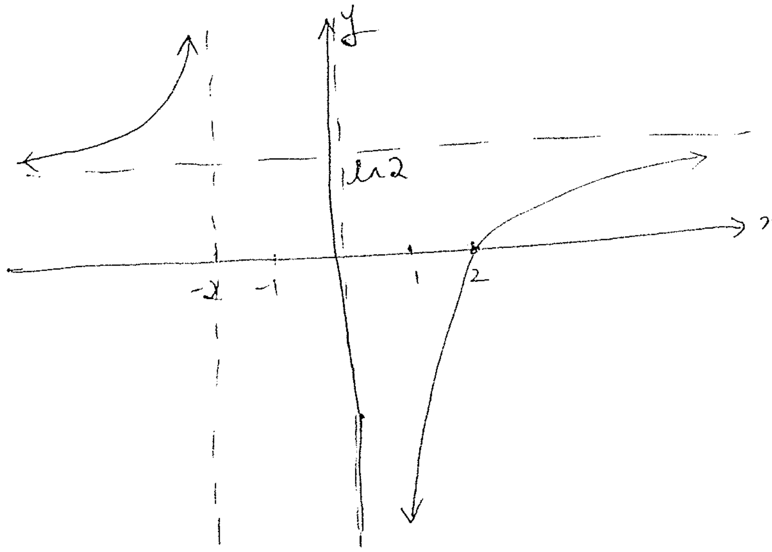
$-4(1+x) = 0$

$\therefore x = -1.$

But $x=-1$ is out of domain

$\therefore$ No inflexion points.

(v)



As $x \to \pm\infty$: $y = \ln 2x - \ln(2+x)$

$= \ln 2 + \underbrace{\ln x - \ln(2+x)}$ regardless of x this cancels each other out.

As $x \to \pm\infty \Rightarrow \ln 2$